6.S098: Introduction to Applied Convex Optimization
IAP 2022

Instructor Information

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Class Information

Dates: Tues/Thurs January 4th - January 27
Time: 1:00pm – 2:30pm
Classroom: 32-124

Course Description

Optimization is a cornerstone of many disciplines in engineering. Surprisingly, many problems such as balancing bipedal robots, designing your own index fund, and allocating power in the energy grid can be formulated as *convex* optimization problems. Convex optimization is a mature technology which can be used to solve a huge number of applied problems and can be scaled to problems with millions of variables. However, recognizing what can be transformed into a convex optimization problem can be challenging. This course will teach you how to recognize, formulate, and solve these problems.

Lectures will be motivated by realistic case studies from areas where convex optimization is used in practice. We will briefly survey theoretical results in convex analysis, but the majority of the course will focus on formulating and solving problems that come up in practice. Applications will include robotics, signal processing, statistics and machine learning, finance, control, power systems, and other areas based on student interest. This course is designed for advanced undergraduates and beginning graduate students.

Prerequisites: multivariable calculus (18.02), linear algebra (18.06 or 18.061), basic probability, programming, mathematical maturity (e.g., 6.042).

Course Objectives

After this course, you should be able to...

- recognize and formulate convex optimization problems that appear in various fields.
- use open source software to solve these optimization problems.
- decide which solver is best for your problem.

Resources

Textbook: *Convex Optimization*, by Stephen Boyd and Lieven Vandenberghe.

Software: `Convex.jl` or `CVXPY`

Course Schedule

The schedule below is tentative and subject to change.

Week 1:
01/04: **Linear Programming (Alex)**
- Case Study: Shortest path in Graphs
- Case Study: Air Traffic Optimization
- Linear programming standard form.
- Practical Considerations: How to scale your program

01/06: **Beyond Linear Programs: Convexity (Alex)**
- Case Study: Finding arbitrages and risk constrained portfolio optimization
- Convex sets
- Convex functions
- Disciplined Convex Programming (DCP)
- Anatomy of Convex Optimization Software

Week 2:
01/11: **Quadratic and Second Order Cone Programming (Alex)**
- Case Study: How to land a SpaceX rocket
- Case Study: Designing a robot grasp.
- Recognizing convexity in real problems

01/13: **Duality: Why is my problem infeasible? (Benoît)**
- The Lagrange dual problem
- KKT conditions
- Sensitivity analysis and risk
- Minimax Optimization

Week 3:
01/18: **Applications: Statistics & Machine Learning (Benoît)**
- Maximum Likelihood Estimation (MLE) and Maximum A Posteriori Probability (MAP)
- Case Study: Support Vector Machines
- Case Study: Experiment design for System Identification
- Classification problems

01/20: **Applications: Power Systems and Benders decomposition (Benoît)**
- Case Study: Power systems
- Case Study: Stochastic multistage control
- Benders decomposition

Week 4:
01/25: **Advanced Topics I: Hard Optimization Problems (Alex)**
- TBD based on class interest
- Suggested Topic: Semidefinite/Cone Programming
- Case Study: Balancing an Atlas Robot on one leg

01/27: **Advanced Topics II: Hard Optimization Problems (Alex)**
- TBD based on class interst

- Suggested Topic: Mixed Integer Programming or Sums-of-Squares/Polynomial Programming
- Case Study: Air transport optimization Revisited. Covering and Partitioning
- Case Study: Applications of Polynomial Optimization

Grading

This course is offered for 6 units of credit, and the grading is P/D/F. To receive credit, you must get 16 or more points. A minimum of 4 points of attendance and 2 points on the final project are required to pass. The main goal of this course is to learn about optimization and solve some fun problems.

Problem sets	12
Attendance	6
Project	4
Total	22

Problem Sets

There will be three problem sets, assigned on Wednesday and due the subsequent Wednesday. Grading will be very coarse–emphasis is put on understanding the concepts over correct answers. Problem sets will focus on giving students practice formulating convex optimization problems and obtaining familiarity with solving those programs using open source software.

Each problem set will ask you to model and solve convex optimization problems that come from various application areas, which will be chosen based on interests of the class. Problem sets will require use of the Julia or Python programming languages, but no prior experience in either is required. Problem sets are expected to take around 4 hours each.

Project

There will be a course project during the last week of IAP that will allow you to apply convex optimization to a problem of interest. The final product will be a short mathematical description of this problem (akin to the descriptions you see on the problem sets) and a solution. Course grading is designed such that this project can take the place of the final problem set. Presentation of the results to the class are a possibility.

Attendance

Attendance is strongly encouraged and will reflected in your final grade. Attending a lecture in person will result in being awarded 1 point.

At a random point in each lecture, we will provide a survey code for those attending the lecture remotely. Students will have 36 hours to submit this survey code to the course staff to receive 0.75 points of attendance credit.

It is possible to mix and match attendance points. For example, attending 4 lectures in person and 3 remotely will result in full attendance credit.

Acknowledgements

Much of the material for this course comes from the following courses:

- Stephen Boyd's Convex Optimization I (link)
- Lieven Vandenberghe's Convex Optimization (link)
- Ryan Tibshirani's Convex Optimization (link)