6.S098 IAP 2022 Final Project

1 Project Description

Arguably the biggest strength of convex optimization is its ability to scale. In terms of speed, solving convex optimization problems is fast enough to be used as a subroutine in other algorithms such as branch and bound. In terms of size, convex optimization scales well enough to be used as a backbone in industrial applications. The purpose of this project is for you to apply convex optimization to a real world problem while exploiting this ability for convex optimization to scale.

Your project must have the following components:

a) Problem Description

- Describe at a high level a problem which occurs in the real world. Do there exist any optimization driven solutions to this problem right now? If so what are it's strengths and weaknesses, if not how could optimization be leveraged to provide a good solution?

b) Problem Data

- What kind of data is involved in the problem you described?
- Where will you get this data? It is strongly encouraged to use as realistic a dataset as possible (preferably a publically avaiable one). Synthetic or random datasets are strongly discouraged and should be checked with the course staff.

c) Problem Solution

- Give a mathematical description of your problem as well as an algorithmic solution to it. Your solution should include one of the following:
  - Solving a very large convex optimization problem.
  - Solving a convex optimization problem repeatedly as a subroutine of your algorithm.
  - Solving many different convex optimization problems.

d) Discussion

- What were the strengths of using convex optimization in your solution? Aspects you might wish to consider are the ability for convex optimization to give globally optimal solutions, certificates of infeasibility, and maturity of current tools for solving optimization problems.
- What were the weaknesses of using convex optimization in your solution? Aspects you might wish to consider are whether you had to use any heuristics in your algorithm, whether the convex problems were surrogates for the true problem you wished to solve, and what kind of relaxations/assumptions you had to make in order to transcribe your problem.

You are not necessarily expected to be completely novel. It is acceptable to read a paper which leverages convex optimization, replicate those results, and discuss.

You may work in teams of up to two. If you do so please include a clear description of the contributions from each team member and know that the expectation for teams is higher than individual projects.

2 Examples of Projects

2.1 QP for Stabilizing Dynamic Locomotion

In An Efficiently Solvable Quadratic Program for Stabilizing Dynamic Locomotion, the authors describe a controller for stabilizing a bipedal robot based on quadratic programming. As the algorithm is meant to control a robot in the real world, solving the quadratic programs at real-time rates is an essential challenge to deploying their solution. Explore the formulation of the dynamic controller, deploy it on a simulated robot, and explore the importance of achieving these real time rates.

2.2 Convex Optimization Layer in a Neural Network

Many convex optimization programs when considered as functions of the program data are differentiable with respect to that data. In Differentiable Convex Optimization Layers, the authors describe how to use CVXPY to act as a non-linear differentiable layer in a neural network. Some of these layers are implemented here. Describe an example problem where it might be advantageous to use a convex optimization layer in your neural network and test this hypothesis on a publically available dataset. For Python, you can use cvxpylayers and for Julia, you can use DiffOpt. Discuss the role of the optimization layer and compare your solution to other architectures.

2.3 Fair Air Traffic Flow Optimization

Efficiently scheduling air traffic is an important part of the modern economy. However, efficiency cannot be the only metric we consider as efficiency can lead to unfairly prioritizing large, established carriers over smaller ones. In this paper, the authors describe a way to balance fairness and efficiency in air traffic optimization. Explore this problem using a real airline dataset which is available upon request to the teaching staff.

2.4 Finding Arbitrages in the Currency Exchange Market

Review this tutorial on finding arbitrages in foreign currency exchange markets. Write a small program to pull in real currency exchange rate data and attempt to find arbitrages in these markets. Report whether you actually find any arbitrages and if so how long they stay open. Since real transactions take time, devise a strategy which might ensure you find arbitrages which can actually be executed.

2.5 Detecting linearity in polyhedral representations

Polyhedra are fundamental objects appearing everywhere in engineering. The feasible sets of LPs and QPs for instance are polyhedra. To view this the other way around, modeling with polyhedra help in having LP or QP formulations that can then be solved efficiently! Polyhedra are defined and described by a finite intersection of halfspaces $\{\, x \in \mathbb{R}^n \mid a^\top x \leq \beta \,\}$. Sometimes, the polyhedra can be inferred to lie in a lower-dimensional space, e.g.,

$$\begin{aligned} x_1 &\geq 0 \\ x_2 &\geq 0 \\ x_1 + x_2 &\leq 0 \end{aligned}$$

implies that $x_1 + x_2 = 0$.

Given a polyhedron defined by m hyperplanes $a_i^\top x \leq \beta_i$, $i = 1, \ldots, m$, we would like to find whether it belongs to a hyperplane. For this, we consider the following LP:

$$\begin{aligned} \min_{\lambda} \sum \lambda_i \alpha_i & \\ \lambda &\geq 0 \\ \sum_{i=1}^{m} a_i \lambda_i &= 0 \\ \sum_{i=1}^{m} \lambda_i &= 1. \end{aligned}$$

What is the dual of this problem ?

What can be concluded from the primal or dual feasibility or infeasibility ? Develop an algorithm that determines whether the polyhedron belongs to a hyperplane and finds the equation of the hyperplane if it does.